Journal of Computational and Applied Mathematics 236 (2011) 812–818

Contents lists available at SciVerse ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Image decoding optimization based on compressive sensing☆

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ARTICLE INFO

Keywords:
Compressive sensing
Image decoding
Total Variation
SOCP

ABSTRACT

Transform-based image codec follows the basic principle: the reconstructed quality is decided by the quantization level. Compressive sensing (CS) breaks the limit and states that sparse signals can be perfectly recovered from incomplete or even corrupted information by solving convex optimization. Under the same acquisition of images, if images are represented sparsely enough, they can be reconstructed more accurately by CS recovery than inverse transform. So, in this paper, we utilize a modified TV operator to enhance image sparse representation and reconstruction accuracy, and we acquire image information from transform coefficients corrupted by quantization noise. We can reconstruct the images by CS recovery instead of inverse transform. A CS-based JPEG decoding scheme is obtained and experimental results demonstrate that the proposed methods significantly improve the PSNR and visual quality of reconstructed images compared with original JPEG decoder.

1. Introduction

In recent years, compressive sensing (CS) [1] has become an important area of signal processing. Due to its interesting practical utility, CS has been widely used in signal compression and image processing. By employing some mathematical programming methods, it is able to reconstruct an originally sparse signal from part of its measurement coefficients. Since a large quantity of image and video frames fit this criterion, increasing amount of research work has been done to incorporate the CS theory into this field.

There are several widely used image compression codec standards for several years, such as JPEG. For using redundancy of pixels to reduce the bit rates, JPEG encoder employs the transform and quantization. Correspondingly, the decoders of them employ the de-quantization and inverse transform to keep the consistency between the encoder side and the decoder side.

For transform operations on the image blocks, we assume a whole image $\mathbb{I}$ of the form:

$$\mathbb{I} = \begin{pmatrix} I^{(11)} & I^{(12)} & \cdots & I^{(1v)} \\ I^{(21)} & I^{(22)} & \cdots & I^{(2v)} \\ \vdots & \vdots & \ddots & \vdots \\ I^{(u1)} & I^{(u2)} & \cdots & I^{(uv)} \end{pmatrix}, \tag{1.1}$$

where $I^{(ij)}$ is its $(i,j)th$ entry of size $n \times n$. Actually, the 2D transform for popular encoder side is as follows.

☆ This paper is supported by the National Basic Research Program of China (973 Program) (No. 2011CB302703), the National Natural Science Foundation of China (No. 61033004, 60825203, 60973056, U0935004), Beijing Natural Science Foundation (4102009).

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doi:10.1016/j.cam.2011.05.023

For input $n \times n$ block I, the 2D forward transform output:

$$b = C \cdot I \cdot R, \tag{1.2}$$

where b is the transform coefficient matrix of size $n \times n$, and the matrix C (respectively R) of size $n \times n$ is column (respectively row) transform matrix.

After the transform, the coefficient matrix b is quantized. For each element b_{ij} of b, the quantization is:

$$W_{ij} = round\,[b_{ij}/q_{ij}], \tag{1.3}$$

where q_{ij} is the element of quantization matrix, and the $round[\cdot]$ function rounds a number to the nearest integer. However, it should be pointed out that there may be different quantization steps for different elements of b, and quantization value W_{ij} will be coded and transmitted to the decoder side.

Correspondingly, the inverse quantization and inverse transform operations can be carried out in the decoder side. The inverse quantization is

$$\bar{b}_{ij} = W_{ij} * q_{ij}, \tag{1.4}$$

where $\bar{b}$ is the transform coefficient with quantization noise and these $\bar{b}_{ij}$ compose $\bar{b}$.

Obviously, quantization noise is given by

$$\bar{b}_{ij} = b_{ij} + e_{ij}. \tag{1.5}$$

The inverse transform can recover the image block from $\bar{b}$, and the reconstructed image block is:

$$\tilde{I} = C^{-1} \cdot \bar{b} \cdot R^{-1}. \tag{1.6}$$

JPEG is based on transform, so that they cannot avoid the limit of transform-based codec: the accuracy of reconstruction must match the quantization level. In other words, the loss of reconstruction accuracy is proportional to the loss of quantization. At first glance, accurately recovering images under the condition of strong quantization noise power appear hopeless. Fortunately, compressive sensing theory says that if an image is sparse, in the sense that it can be written either exactly or accurately as a superposition of a small number of vectors in some fixed basis, accurate and sometimes exact recovery can occur by solving a convex optimization problem.

There are several studies about CS theory and its applications. [2] and [3] focused on CS theory research. [4–6] gave different proposals to enhance the performance of image coding. For video coding, [7] gave a new H.264 framework based on the CS theory, especially for images or video objects containing edges.

In our research, we focus on enhancing the performance of image decoding through CS theory. Considering the practical utility, we just employ the CS reconstruction combination blocks in the JPEG decoder to replace the inverse transform. Due to the robust character of CS theory, it can highly upgrade the image quality over original JPEG reconstruction.

The rest of this paper is organized as follows. The CS theory will be introduced in brief in Section 2. Our proposed decoding method including the sparse basis matrixes selection and blocks joint reconstruction will be presented in Section 3. The experiment results will be showed in Section 4. Section 5 concludes this paper.

2. Brief introduction of compressive sensing

The compressive sensing, which is also called compressive sampling, employs non-adaptive linear projections that preserve the structure of the signal; the signal is then reconstructed from these projections using a programming optimization process.

Suppose x is an unknown vector in $\mathbb{R}^N$, we plan to reconstruct it from the following acquisition system

$$y = \Phi x, \tag{2.1}$$

where Φ is an $M \times N (M < N)$ measurement matrix. In that case, the measurement y of the original signal is an $M \times 1$ vector. In general, solving the under-determined system appears hopeless, as it is easy to make up examples for which it clearly cannot be done. But, suppose we know a priori knowledge that x has sparse representation in some domain (e.g. basis, frame), it is possible to reconstruct it from (2.1).

Let signal $x \in \mathbb{R}^N$ has decomposition as

$$x = \sum_{i=1}^{N} \varphi_i s_i \tag{2.2}$$

or

$$x = \Psi s, \tag{2.3}$$

where $\Psi = \{\varphi_1, \varphi_2, \ldots, \varphi_N\}$ with the vectors $\{\varphi_i\}$ as columns, and s is the coefficients of x in Ψ-domain. We say x is sparse in the Ψ-domain if the coefficients sequence is supported on a small set, i.e., most of its coefficients are zero. If x is a time or space domain signal, s is a Ψ domain signal. In other words, x and s are equivalent representations of the same signal. That means the signal x has an interesting sparse representation s, which has only K non-zero coefficients and $(N-K)$ zero coefficients.

The CS theory says that when $M \geqslant cK \log N$, where c is a positive number, the signal s can be reconstructed exactly by solving the following minimum l_1-norm optimization problem [8]:

$$\tilde{s} = \arg\min \parallel s \parallel_{l_l}, \quad \text{s.t. } y = \Theta s, \tag{2.4}$$

where $\Theta = \Phi\Psi$ [9].

However, in any realistic application, we cannot expect to measure Φx without any error. Consider the following acquisition system

$$y = \Phi x + e, \tag{2.5}$$

where e represents the acquisition noise. The x can be recovered by solving:

$$\tilde{x} = \arg\min \parallel \Psi^T x \parallel_{l_l}, \quad \text{s.t. } \parallel \Phi x - y \parallel_{l_2} \le \varepsilon, \tag{2.6}$$

where ε is an error caused by e.

This convex optimization problem can be solved by several methods, such as Iterative Shrinkage/Thresholding (IST) [10], Gradient Projection (GP) [11], Matching Pursuit (MP) [12]and Projection onto Convex Sets (POCS) [13,14].

Small perturbations in the observed data should induce small perturbations in the reconstruction. Fortunately, the recovery procedures may be adapted to be surprisingly stable and robust over arbitrary perturbations [2].

3. Proposed image decoding method

3.1. Image reconstruction based on CS

Due to the specialty of CS theory, we propose that the transform of the encoder side could be regarded as a CS measure step. Naturally, the quantization noise in the encoder side could be regarded as e. Essentially, the acquisition system of the encoder side is similar to the CS observation system (2.5), and the image blocks can be reconstructed by CS recovery instead of inverse transform.

As we all know, in most blocks, the values of pixels are non-zero, and just a small number of blocks can be defined as sparse 2D signal in time domain. However, [15] points out that if the underlying signal is a 2D image, an alternate recovery model is that the gradient is sparse. Based on [16], which shows that the gradient operator can be used in CS reconstruction, we present a modified total variation (TV) method. Let I_{ij} represent the pixel in the ith row and jth column of an $n \times n$ image block I. Then we define the horizontal operator and the vertical operator as

$$D_{h;ij} I = \begin{cases} I_{i+1,j} - I_{ij} & i < n \\ I_{i-1,j} - I_{ij} & i = n, \end{cases} \tag{3.1}$$

$$D_{v;ij} I = \begin{cases} I_{i,j+1} - I_{ij} & j < n \\ I_{i,j-1} - I_{ij} & j = n, \end{cases} \tag{3.2}$$

and $D_{ij}I$, a kind of discrete gradient of I, is defined by

$$D_{ij} I = \begin{pmatrix} D_{h;ij} I \\ D_{v;ij} I \end{pmatrix}. \tag{3.3}$$

So the modified total variation of image I is:

$$TV(I) = \sum_{ij} \sqrt{(D_{h;ij}I)^2 + (D_{v;ij}I)^2} = \sum_{ij} \|D_{ij}I\|_2. \tag{3.4}$$

In this case, we can recover the image block I from the noise measurements by solving the program:

$$\tilde{I} = \arg\min TV(I), \quad \text{s.t. } \parallel \Phi I - y \parallel_2 \le \varepsilon. \tag{3.5}$$

Considering the particularity of common image codec, which uses the 2D transform, it is obvious that a transform such as DCT could be regarded as observation, and the quantization noise could be error. However, for the purpose of simplicity and accordance with Eq. (2.3), we need to change the 2D transform in image codec to equivalent 1D transform. Let $N = n \times n$, the $n \times n$ image signal I could be reformed to $N \times 1$ vector x through column-by-column scan. Correspondingly, the transform coefficient matrix $b_{n\times n}$ should also be reshaped from matrix to vector $y_{N\times 1}$ through column-by-column scan. Hence, the transform in (1.2) is replaced by the new transform, which is given by

$$y_{N\times 1} = A_{N\times N} \cdot x_{N\times 1}. \tag{3.6}$$

$A_{N\times N}$ is equivalent to $(C_{n\times n}, R_{n\times n})$ for signal $I_{n\times n}$, which has been reshaped to vector $x_{N\times 1}$ in (3.5). To calculate $A_{N\times N}$, we recast element b_{ij}:

$$\begin{aligned} b_{ij} &= \sum_l C_{il} \sum_k I_{lk} R_{kj} \\ &= \sum_l \sum_k C_{il} R_{kj} I_{lk} \\ &= \sum_{lk} (C_{il} R_{kj}) I_{lk}. \end{aligned} \tag{3.7}$$

So (c, d) element of $A_{N\times N}$ is $A_{cd} = C_{il}R_{kj}$, where $c = n(j-1)+i$ and $d = n(l-1)+k$.

Due to column-by-column scan, the horizontal operator will be recast as follows.

$$\hat{D}_{h;ij}x = \begin{cases} x_{n(j-1)+i+1} - x_{n(j-1)+i} & i < n \\ x_{n(j-1)+i-1} - x_{n(j-1)+i} & i = n \end{cases} \tag{3.8}$$

The vertical operator is

$$\hat{D}_{v;ij}x = \begin{cases} x_{nj+i} - x_{n(j-1)+i} & j < n \\ x_{n(j-2)+i} - x_{n(j-1)+i} & j = n. \end{cases} \tag{3.9}$$

So the modified total variations of reshaped image x is:

$$\hat{TV}(x) = \sum_{ij} \sqrt{(\hat{D}_{h;ij}x)^2 + (\hat{D}_{v;ij}x)^2} = \sum_{ij} \|\hat{D}_{ij}x\|_2. \tag{3.10}$$

Thus, the optimization problem should be:

$$\tilde{x} = \arg\min \hat{TV}(x), \quad \text{s.t. } \| Ax - y \|_2 \le \varepsilon, \tag{3.11}$$

where $\tilde{x}$ is the reconstruction of x. It can be reshaped to $n \times n$ matrix $\tilde{I}_{n\times n}$ as reconstruction of image signal.

3.2. Block combination optimization recovery

It is well known that the image texture is continuous while image codec is based on image block. The division of image breaks down the continuity of texture, so semi-norm $\hat{TV}(\cdot)$ cannot work well over small dimension block. Because the sparsity of $\hat{D}_{h;ij}x$ and $\hat{D}_{v;ij}x$ impact on the quality of CS reconstruction, the so-called blocks combination optimization reconstruction can be illustrated as follows.

First, rewrite the image $\mathbb{I}$ in the form:

$$\mathbb{I} = \begin{pmatrix} J^{(11)} & J^{(12)} & \cdots & J^{(1V)} \\ J^{(21)} & J^{(22)} & \cdots & J^{(2V)} \\ \vdots & \vdots & \ddots & \vdots \\ J^{(U1)} & J^{(U2)} & \cdots & J^{(UV)} \end{pmatrix}, \tag{3.12}$$

where the $J^{(ij)}$ is the big image block which is combined by the small block $I^{(ij)}$. For ease of presentation, suppose that the number of small blocks, which will be combined to a big block, is p^2. In such a way, it is natural that

$$J^{(11)} = \begin{pmatrix} I^{(11)} & I^{(12)} & \cdots & I^{(1p)} \\ I^{(21)} & I^{(22)} & \cdots & I^{(2p)} \\ \vdots & \vdots & \ddots & \vdots \\ I^{(p1)} & I^{(p2)} & \cdots & I^{(pp)} \end{pmatrix}, \tag{3.13}$$

and

$$J^{(ij)} = \begin{pmatrix} I^{(p(i-1)+1,p(j-1)+1)} & I^{(p(i-1)+1,p(j-1)+2)} & \cdots & I^{(p(i-1)+1,pj)} \\ I^{(p(i-1)+2,p(j-1)+1)} & I^{(p(i-1)+2,p(j-1)+2)} & \cdots & I^{(p(i-1)+2,pj)} \\ \vdots & \vdots & \ddots & \vdots \\ I^{(pi,p(j-1)+1)} & I^{(pi,p(j-1)+2)} & \cdots & I^{(pi,pj)} \end{pmatrix}. \tag{3.14}$$

Thus, the transform or measure in the encoder could be regarded as

$$B^{(ij)} = \begin{pmatrix} C & 0 & \cdots & 0 \\ 0 & C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C \end{pmatrix} \cdot J^{(ij)} \cdot \begin{pmatrix} R & 0 & \cdots & 0 \\ 0 & R & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R \end{pmatrix}, \tag{3.15}$$

where $B^{(ij)}$ is transform coefficients matrix of $J^{(ij)}$. Similarly, (3.14) can be also reformed to a new form of transform

$$Y_{Np^2\times 1} = \mathbb{A}_{Np^2\times Np^2} \cdot X_{Np^2\times 1} \tag{3.16}$$

where vector X is reshaped from J, vector Y is reshaped from B, and $\mathbb{A}$ is a 1D transform matrix.

With all the above specifications, the proposed optimization problem should be:

$$\tilde{X} = \arg\min \hat{TV}(X), \quad \text{s.t. } \| \mathbb{A}X - Y \|_2 \le \sigma, \tag{3.17}$$

where σ depends on quantization noise.

Practically, (3.16) can be written in the form:

$$\min \sum_{ij} t_{ij}, \quad \text{s.t. } \begin{cases} \|\hat{D}_{ij}X\|_2 \le t_{ij}, & i,j = 1\dots n \\ \|\mathbb{A}X - Y\|_2 \le \sigma. \end{cases} \tag{3.18}$$

Setting

$$\begin{cases} f_{ij} = \dfrac{1}{2}(\|\hat{D}_{ij}\|_2^2 - t_{ij}^2), & i,j = 1\dots n \\ f_\sigma = \dfrac{1}{2}(\|\mathbb{A}X - Y\|_2^2 - \sigma^2) & \\ z = \begin{pmatrix} X \\ t \end{pmatrix} & \end{cases}, \tag{3.19}$$

where t is a vector formed by t_{ij}. We can rewrite the problem (3.18) as:

$$\min_z \langle c_0, z\rangle \quad \text{s.t. } \begin{cases} f_\sigma(z) \le 0 \\ f_{ij}(z) \le 0, \quad i,j = 1\dots n, \end{cases} \tag{3.20}$$

and (3.20) can be recast to an SOCP problem.

For normal SOCP problem:

$$\min_z \langle c_0, z\rangle, \quad \text{s.t. } \begin{cases} \|\mathbb{A}z - Y\|_2 \le \sigma \\ f_i(z) = \dfrac{1}{2}[\|\mathbb{A}_i z\|_2^2 - (\langle c_i z\rangle + d_i)^2] \le 0, \end{cases} \tag{3.21}$$

it can be solved by log-barrier method:

$$\begin{cases} \min_z \langle c_0, z\rangle + \dfrac{1}{\tau^k}\sum_i [-\log(-f_i(z))], \quad \text{s.t. } \|\mathbb{A}_0 z - Y\|_2 \le \sigma \\ \tau^k > \tau^{k-1} \quad (\tau^k = \mu\tau^{k-1}), \end{cases} \tag{3.22}$$

and every step can be solved by Newton iteration.

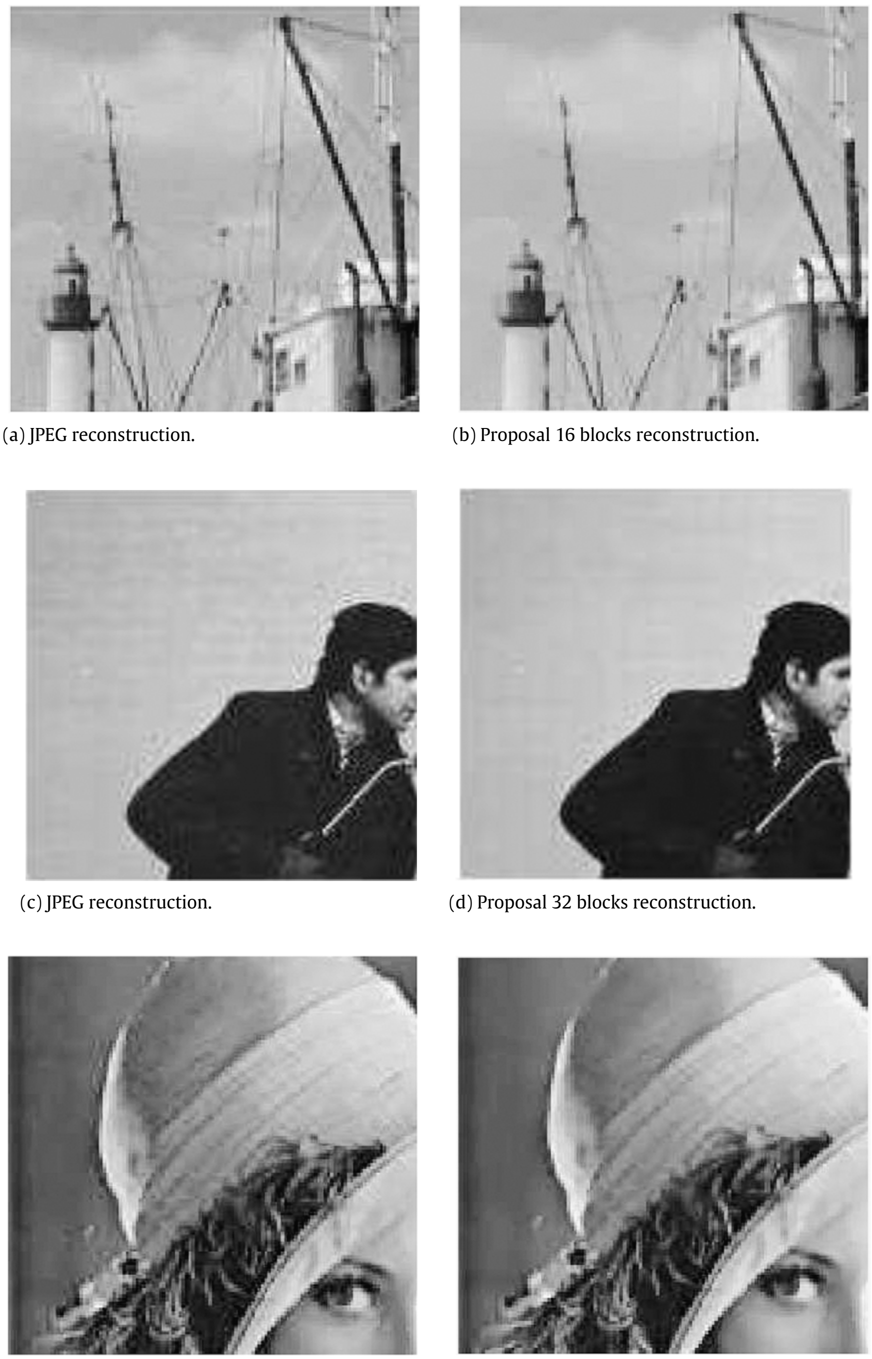
Finally, the $\tilde{X}$ will be reshaped to $\tilde{J}_{pn\times pn}$ as reconstruction instead of inverse transform recovery.

4. Experimental results

The performance of proposed algorithms is provided in this section. The proposal in this paper has been integrated into JPEG decoder. The block dimension in JPEG is 8×8; meanwhile, we also use 16 8×8 blocks to solve together and output a 32×32 block. In addition, 4 blocks are used to solve together and to output a 16×16 block.

Quantization error is another important parameter. We estimate several different values, and at last we confirm the best value of σ, which can get the best reconstruction through experiments. The final experimental results are shown in Table 1.

Left images of each pair are reconstructed by original JPEG. For the right ones, (a) and (b) are 4 8×8 blocks to solve together and to output a 16×16 block, (c) and (d) are 16 8×8 blocks to solve together and to output a 32×32 block.



(a) JPEG reconstruction.

(b) Proposal 16 blocks reconstruction.

(c) JPEG reconstruction.

(d) Proposal 32 blocks reconstruction.

(e) JPEG reconstruction.

(f) Proposal 32 blocks reconstruction.

Fig. 1. Reconstructions of images.

It is obvious that using CS reconstruction can get significant PSNR gain. The average PSNR of outputting a 32×32 block with CS reconstruction can get more than 0.5 db gain. Meanwhile, the subjective quality also has obvious upgrade, especially at the edge of objects, such as in Fig. 1, because the CS recovery criterion is based on a minimum sum of absolute pixel value criterion, and blocks containing simple edge can be better reconstructed in this approach [7].

In our experiments, a shortcoming of SOCP process in CS reconstruction is its high computational complexity. With large number of steps of iteration, it takes several minutes to decode a 256×256 image. Practically, we can combine more blocks to reconstruct jointly till the whole image is recovered at a stroke. We can imagine that if we do so, the PSNR gain will be

Table 1
The PSNR (db) of different images with different reconstruction methods.

Images	Original JPEG	CS 8×8 $\sigma = 10$	CS 16×16 $\sigma = 24$	CS 32×32 $\sigma = 50$
barche256 $\times$ 256	31.7827	31.9997	32.1545	32.2441
camera256 $\times$ 256	31.6328	31.8229	31.9654	32.0762
lena256 $\times$ 256	32.8970	33.0782	33.2514	33.3830
peppers256 $\times$ 256	33.0834	33.3189	33.5559	33.7295
average	32.3490	32.5549	32.7318	32.8582

the largest with the highest complexity. Therefore, we should study in depth to keep balance between the performance and efficiency.

5. Conclusion

This paper focuses on the image coding scheme which can be improved by CS theory. To upgrade the quality of CS reconstruction, the proposal of this paper improves optimization theory on CS area, include gradient sparse operators and CS optimized reconstruction with combination blocks. The advantage of this algorithm is that no change is needed on the encoder side and the improvement will be focused on the decoder side. This design is of great significance to the application of image reconstruction, because this method can get better effects during decoding the image under the existing image compressive standards such as JPEG. In the future, the more effective sparse operators will be studied to enhance the reconstruction quality and an error model will be contributed. Moreover, the CS theory itself can be used in many fields other than image codec.

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